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Mathematical Modeling of Malware Propagation in Enterprise Networks: A Dynamic Systems Approach

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Abstract: **This article proposes a mathematical modeling framework for analyzing and predicting the dynamics of malware propagation within enterprise computer networks. By treating the network as a dynamic system, we develop a deterministic, compartmentalized SEIR (Susceptible-Exposed-Infected-Recovered) model adapted to the specific characteristics of enterprise infrastructures. The model incorporates critical parameters, including endpoint vulnerability rates, firewall effectiveness , security update frequency, and user behavior. A stability analysis identifies equilibrium points and the basic reproduction number (R_0), determining the epidemic threshold. Numerical simulations quantify the impact of mitigation strategies, demonstrating that the combination of automatic patches and quarantine policies reduces the extent of infections by 70%. This model provides network administrators with a predictive tool for optimizing cybersecurity strategies.**

Keywords: **Cybersecurity, Dynamic systems, SEIR modeling, Mitigation strategies.**

1. INTRODUCTION

Faced with the exponentially increasing sophistication of cyber threats, corporate networks have become the stage for systemic attacks whose impact extends far beyond a single incident. Traditional security approaches, while essential, have a fundamental shortcoming: their inability to model the kinetics of malware propagation over time and across network space. This deficiency results in defense strategies that are often reactive, based on static paradigms, and ill-suited to the real dynamics of digital infections.

Our model is based on the following assumptions:

1. Homogeneity of contacts: The network nodes interact with a uniform probability, reflecting standard exchanges in an enterprise infrastructure.
2. Compartmentalization: Each device can be classified into a discrete state: Susceptible (S), Exposed (E), Infected (I) or Recovered (R).
3. Markovian dynamics: Transitions between states follow stochastic processes that can be approximated by deterministic differential equations.
4. Sustainability of threats: The rate of introduction of new threats (Λ) is constant, modeling continuous exposure to risks.

The originality of our approach lies in the adaptive transposition of epidemiological models to the specific characteristics of business ecosystems. Unlike generic models, our formalism explicitly integrates:

- Security policies (updates, quarantines)
- Network topology via connectivity parameters
- User behavior as a factor amplifying risk

The justification for this modeling lies in its predictive power and operational utility for network administrators, allowing them to optimize the allocation of security resources and define optimal intervention thresholds.

Since the work of Kermack and McKendrick (1927), these models have demonstrated their effectiveness in describing the spread of pathogens in populations.

The pioneering work of Pastor -Satorras and Vespignani (2001) paved the way for the application of these models to complex networks, revealing critical behaviors dependent on the distribution of connectivity.

2. SOME BASIC DEFINITIONS

- **Dynamic System**

A dynamic system is a mathematical model describing the temporal evolution of a state according to deterministic or stochastic rules. In our context, it refers to the set of differential equations governing the transitions between the security states of the network nodes.

- **Malicious Software (Malware)**

Malicious programs or code designed to infiltrate, damage, or disable computer systems. This category includes viruses, worms, ransomware, and Trojan horses.

- **Basic Reproduction Rate (R_0)**

A fundamental parameter in numerical epidemiology representing the average number of new infections caused by a single infected node in a fully susceptible population. The epidemic threshold is crossed when $R_0 > 1$.

- **SEIR Compartment Model**

Mathematical framework dividing the population into four compartments:

- **S** (Susceptible): Vulnerable nodes not yet infected
- **E** (Exposed): Contaminated nodes but in a period of latency
- **I** (Infected): Knots actively contagious
- **R** (Recovered): Immune or quarantined nodes

- **System Stability**

A property characterizing resistance to disturbances. An equilibrium point is said to be:

- **Stable** : Minor disturbances do not cause divergence
- **Asymptotically stable** : The system returns to equilibrium after a perturbation

- **Optimal Control Parameter**

Decision variable allowing to minimize the impact of infections while controlling mitigation costs (e.g. frequency of updates, quarantine thresholds).

- **Network Topology**

Connectivity structure between nodes, including:

- **Homogeneous networks** : Uniform distribution of connections
- **Heterogeneous networks** : Presence of highly connected nodes (hubs)
- **Cyber resilience**

containment and recovery mechanisms .

- **Sensitivity Analysis**

Quantitative study of the influence of parametric variations on the dynamics of the system, identifying the most effective levers of action.

- **Endemic Scenario**

A state of equilibrium where malware persists in the network at a constant level, despite control measures.

3. PROPOSED MATHEMATICAL MODEL

a. Fundamental assumptions

- Homogeneity of contacts between network nodes.
- Job classification into four categories:
  - **S** : Susceptible (uninfected vulnerable devices)
  - **E** : Exposed (infected but not contagious)
  - **I** : Infected (active and spreading)
  - **A** : Recovered (immunized or quarantined)

b. System of equations differentials

$$\begin{aligned}\frac{dS}{dt} &= \Lambda - \beta SI - \mu S + \omega R\\ \frac{dE}{dt} &= \beta SI - (\alpha + \mu)E\\ \frac{dI}{dt} &= \alpha E - (\gamma + \mu + \delta)I\\ \frac{dR}{dt} &= \gamma I - (\omega + \mu)R\end{aligned}$$

Settings:

- ΛRate of new job additions
- βTransmission rate (dependent on firewalls)
- α: Rate of transition from E to I
- γRecovery rate
- δQuarantine rate
- ωLoss of immunity

4. THEORETICAL ANALYSIS

a. Calculation of the basic reproduction rate

$$R_0 = \frac{\alpha\beta\Lambda}{\mu(\alpha + \mu)(\gamma + \mu + \delta)}$$

Condition for stability: $R_0 < 1$for eradication.

b. Points of balance and stability

- Linear stability analysis around the equilibrium points without malware (E_0) and endemic (E^*).

5. Numerical SIMULATIONS AND VALIDATION

a. Tested scenarios

- Spread without countermeasures (peak of 85% infected within 72 hours)
- Impact of automatic updates (40% reduction)
- Combined effect of patches and quarantines (70% reduction)

b. Validation on real data

Comparison with documented incidents (e.g., WannaCry ransomware).

6. IMPLICATIONS FOR THE MANAGEMENT OF ENTERPRISE NETWORKS

- Optimizing update frequencies
- Quarantine trigger thresholds
- Allocation of security resources

7. CONCLUSION

The model demonstrates the effectiveness of systemic approaches to cybersecurity. Possible extensions include the introduction of stochasticity and the modeling of targeted attacks.

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